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THE PROCESS OF THE DEVELOPMENT OF A TOOLKIT TO SUPPORT PRE-SERVICE TEACHERS' UNDERSTANDING AND PRESENTATION OF LENGTH IN MATHEMATICS CLASSROOM

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South African learners perform poorly in mathematics, throughout all the school phases. The training of teachers, especially in the primary school level, has been identified as one of the contributing factors. When pre-service teachers leave training institutions, they are not adequately prepared to teach the mathematical content in the grades for which they have been trained. Added to this is the challenge of insufficient or inadequate material for them to competently teach various topics in the mathematics curriculum. The discrepancy in the material used by South African teacher training institutions to prepare the primary school teachers exacerbates the challenge and calls for the development of relevant material to facilitate the teaching of mathematics with understanding. This paper reports on the process of developing a Toolkit for teaching length to Intermediate Phase learners. Recommendations are also made on possible steps to follow, for other teachers to develop similar Toolkits.

INTRODUCTION AND BACKGROUND

The South African education system has been faced with many challenges, particularly in the context of the teaching and learning of mathematics. For example, teachers do not have sufficient mathematical content knowledge for the level of schooling they are supposed to teach (Bernstein, 2013; Spaull, 2013; Luneta, 2014). Also, teachers use procedural methods of teaching that do not promote conceptual and connected mathematical understanding (Graven & Venkat, 2017). Other challenges involve the shortage of relevant teaching and learning resources (Modisaotsile, 2012) for developing learners' understanding of mathematical concepts.

The training and development of teachers in Initial Teacher Education (ITE) programmes in South Africa has been identified as one of the areas that contribute to some of the challenges mentioned above. For example, despite the guidelines stipulated by the Department of Higher Education and Training (DHET) regarding the teaching and learning of mathematics, different universities differ in the way they interpret and implement these guidelines (Fonseca, Maseko & Roberts, 2018). Furthermore, mathematics courses offered to pre-service teachers (PST's) at different universities in South Africa vary in terms of the mathematics content dealt with. Some universities offer courses with advanced mathematics content, others empower PST's with

mathematics at the grade level in which they will teach, while some focus on the mathematical thinking that teachers need to do (Ball, Hill & Bass, 2005).

In an effort to address the disparity between the ITE programmes in South African universities, the DHET then initiated a four year (2017 – 2020) Primary Teacher Education (PrimTEd) project, as part of its Teaching and Learning Development Capacity Improvement Programme (TLDCIP). The initial work done through the project culminated in the formation of several working groups (WG's), each dealing with a different focus area, such as Literacy/Reading, Number sense, Geometry and Measurement, Mathematical thinking, Knowledge management, Assessment and Work-Integrated Learning (WIL).

According to the PrimTEd project plan, four deliverables, namely, teaching standards curriculum frameworks, materials and assessment tools, need to be produced for the ultimate implementation by all South African universities (PrimTEd Newsletter, October, 2016). The Geometry and Measurement Working Group decided upon the approach of developing resources in the form of Toolkits for teaching various content topics in the Intermediate Phase (IP). This paper reports on the development of the Toolkits in the content area of Length.

The main research question for this study is:

1. What are the provisions in the CAPS document to help with the teaching and learning of the concept of length?
2. What materials are needed to support the teaching and learning of length?
3. What are the processes in the development of material to support the teaching and learning of length?

Pre-service Mathematics teacher education in South Africa

Despite the fact that mathematics education is regarded as one of the national priorities in South Africa, learners continue to be offered inferior mathematics education (Stols, et. al., 2015). The resulting poor performance of South African learners in several Trends in International Mathematics and Science Studies (TIMSS's) is evidence that there is a crisis in South African mathematics education (Spaull, 2013). Whereas the quality of teaching depends on the teachers' knowledge that they bring to the classroom (Ball, Hill & Bass, 2005; Charalambous, 2011; Rowland & Ruthven, 2011), teacher training in mathematics seems to be inadequate in that teachers lack mathematical content knowledge (MCK) as well as the skills required to present effective lessons. Consequently, learners suffer, because they tend to passively rely on the teacher for their success in mathematics, particularly, South African learners, instead of their own independent thinking and sense making of mathematics (Graven, Hewana & Stott, 2013). Therefore, if PST's are to be prepared adequately, teacher training should not only develop their MCK, but their content knowledge for teaching (CKT – M).

According to Hill *et. al.* (2008, p. 431), mathematical knowledge for teaching involves more than just the knowledge of the subject matter that is required to teach the subject,

but also "the subject matter knowledge that supports that teaching. For example, why and how specific mathematical procedures work, how best to define a mathematical term for a particular grade level, and the types of errors students are likely to make with particular content". Pre-service teacher education should therefore prepare and produce PST's who are empowered with both the knowledge of concepts to be taught, as well as methods on how to implement processes that will result in quality instruction in their mathematics classrooms. The development of the IP Toolkit on length, for use by PST's during their WIL sessions and beyond, is one way of providing PST's with knowledge and skills to enable them to effectively deliver the mathematics curriculum in their classes.

Length as an attribute of measurement

One of the real-world applications of mathematics is measurement, which links two components of mathematics, namely, geometry and real numbers (Clements, 1999). Measurement involves assigning a number to indicate a comparison between the attribute of an object being measured and the same attribute of a given unit of measure (Van de Walle, Karp & Bay-Williams, 2015).

Length, as one of the attributes of measurement, involves measuring the distance between two points or the straight line between them (Sarama & Clements, 2009; Suggate, Davis & Gouding, 2010). Several big ideas or foundational concepts that underlie linear measurement have been identified. These include unit iteration, accumulation of distance, conservation, additivity, equal partitioning, transitivity and relation between number and measurement (Sarama & Clements, 2009; Van de Walle *et. al.*, 2015; Tan-Sisman & Aksu, 2016; Hansen, 2017). These ideas form part of the Toolkit that we developed and will be discussed later in this paper.

The South African Curriculum and Assessment Policy Statement (CAPS) on mathematics in the Intermediate Phase stipulates the requirements for teaching the concept of length. The main concepts and skills that learners in the IP need to master are: estimation and practical measurement of length using appropriate measuring instruments and formal units of length; recording, comparing and ordering lengths, in different contexts; calculations and problem-solving involving length, including conversions between different units of length (DBE, 2011a). The big ideas and key concepts involved in the measurement of length, as mentioned in the preceding paragraph, have to be applied during the teaching and learning of the length concepts as prescribed in the CAPS. However, most of these big ideas and concepts are not explicit in this curriculum document. Hence a Toolkit on length has to fill in these gaps by including such concepts.

PST's need to be empowered with skills to address learners' errors and misconceptions when dealing with the concept of length. Research shows that learners find the measurement of length challenging. For example, they tend to leave gaps between units or overlap units when they perform the iteration procedure (Lehrer, 2003). Other errors made by learners include the incorrect alignment of the measuring instrument with the

object whose length is being measured (Tan-Sisman & Aksu, 2016). They also read the scale incorrectly by, for example, counting the number of marks on the instrument, instead of the number of spatial intervals in the distance spanned by length (Solomon & Vasilyeva, 2015). Errors and misconceptions in the measurement of length do not feature in the CAPS and this gap is identified and addressed in the Toolkit we developed.

The development of a Toolkit on teaching the topic of length in the IP

The Toolkits for teaching length in Grades 4, 5 and 6, were designed to cover, as a minimum, concepts and skills as stipulated in the South African CAPS for the IP. The aim of the Toolkits is to provide guidance with sets of materials in the form of content and pedagogical guidelines, which pre-service teachers can use in order to, firstly, understand themselves the key concepts and theory involved in the topic, and secondly, know how to teach the topic competently to IP learners. Therefore the Toolkits consist of CKT-M and MCK on length, as well as Pedagogical Content Knowledge (PCK) involved in the measurement of length. PCK "bridges content knowledge and the practice of teaching, ensuring that discussions of content are relevant to teaching and that discussions of teaching retain attention to content" (Ball, Thames & Phelps, 2008, p. 3).

The components of the Toolkit on length are shown in Table 1 below.

TABLE 1: The components of the Toolkit on length

Section A	Section B
1. Theory around the concept of length: • Definitions pertaining to the measurement of length. • Explanation of related terms and concepts. 2. CAPS requirements on the topic, for the three grades. 3. Big ideas and key concepts in the measurement of length. 4. Errors and misconceptions associated with the teaching and learning of length.	1. Focus areas of each sub-topic of length, as stipulated in CAPS. 2. Learners' prior knowledge, linked to each focus area. 3. An introduction for each sub-topic. 4. Activities to be done with learners to facilitate mastery of concepts and skills. 5. Formative assessment for each activity; possible solutions. 6. Summative assessment for each sub-topic; possible solutions. 7. Clarification notes and teaching guidelines. 8. List of resources used for each sub-topic.

Conceptually oriented and procedurally oriented tasks have been included purposefully in dealing with the topic of length in the Toolkits. Procedurally oriented tasks are

routine tasks whereby previously learned step-by-step solution methods or mathematical computations are utilized, whereas conceptually oriented tasks involve the use of non-routine and original tasks that require understanding of fundamental principles or concepts (Tan-Sisman & Aksu, 2016). A gap seems to exist in the current DBE curriculum material in that conceptual understand of various sub-topics on length is not explicitly specified in the CAPS, nor is it adequately covered for most sub-topics in the DBE Workbooks. For example, in the Grade 4 Workbook, quite a number of questions and tasks on length are procedural in nature, with instructions such as: "Measure each object and give your answer in cm and mm. Order the objects from shortest to longest", or "Complete the numbers on the ruler, measure the lines and complete the table" (Department of Basic Education 2019, pp. 110 – 111). Both tasks in this case involve the procedure of aligning the measuring instrument with the object or line whose length is being measured, without much demand for conceptual understanding. Thus deep mathematical thinking is not encouraged.

In the Grade 4 Toolkit, a conceptually oriented task, comparable to the tasks given above, appears in Figure 1a below, with the following instruction:

> "Your friends, Thando and Lucy, have a disagreement about the height of the picture of the tree in the diagram below. Thando says that the picture has a height of 3 cm while Lucy says it has a height of 8 cm. Which one, if any, of your friends is correct? Explain your answer".

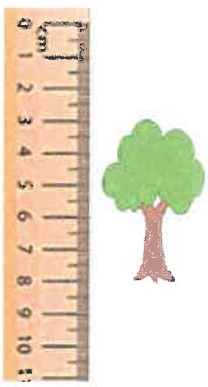

FIGURE 1a: Conceptually oriented task

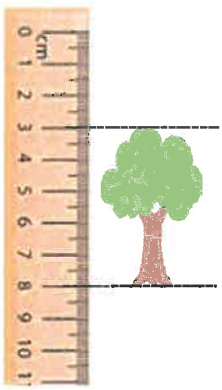

FIGURE 1b: Possible solution strategy

In the above task, learners are encouraged to think about why the height of 3 cm or 8 cm, or both might be wrong. In the process they intuitively use the fundamental principles of accumulation of distance and additivity. That is, they need to understand that "as you iterate a unit along the length of an object and count the iteration, the number words signify the space covered by all units counted up to that point" (Sarama & Clements 2009, p. 276). Therefore they would realize that the height of the picture does not span over 3 units (cm) or 8 units (cm). Hence they would have to find correct ways of determining the height, such as "to subtract the initial measurement from the final measurement" (DBE 2011a, p. 73). Alternatively, they would have to begin counting the iteration from any origin through the interval spanned by the height, up to

the other endpoint. They might even have to think of drawing a line to make it easier for themselves to read the measurements, as depicted in Figure 1b above.

Another shortcoming with the DBE CAPS and Workbooks is the explicit incorporation of learners' possible errors and misconceptions in the teaching and learning of the concept of length. Yet errors and misconceptions can be used as catalysts for learning, whereby learners' mathematical understanding is developed through identifying and addressing their errors (Tan-Sisman & Aksu, 2016). Therefore it is suggested that errors and misconceptions should be included in the mathematics curriculum to inform teachers beforehand about learning challenges that they can expect (Ryan & Williams, 2007). Hence the Toolkits on length specify possible learners' errors and misconceptions that could be expected, as well as suggestions towards addressing them.

CRITICAL STEPS IN DEVELOPING A TOOLKIT ON LENGTH

Below we outline the steps taken and used as a guide in the development of the Toolkit. Examples of material from the Toolkit are also included to provide clarity.

Analysis of SA curriculum documents

The point of departure in the development of the Toolkit was the analysis of the SA curriculum requirements for teaching the topic of length in the applicable grades. The following documents were consulted: CAPS; *Guidelines for responding to learner diversity in the classroom through CAPS*; *Mathematics Teaching and Learning Framework for South Africa: Teaching mathematics for understanding*; the Workbooks and other Learning and Teaching Support Material (LTSM) for the various IP grades. The concepts, skills and processes involved in mastering the content involving length were identified and gaps were addressed when designing the activities to be done by learners.

The design of activities in each grade Toolkit was based on critical information found in the CAPS as depicted in the table below:

TABLE 2: CAPS information guiding the development of activities in the Toolkits

Grade 4	**Grade 5**	**Grade 6**
Term 2: 7 hours (DBE 2011a, pp. 73–75)	Term 2: 6 hours (DBE 2011a, pp. 163–165)	Term 3: 5 hours (DBE 2011a, pp. 272 – 274)
Activities were based on sub-topics for length that were, in turn, created based on the concepts and skills stipulated on the pages as indicated above. The number of sub-topics for each grade corresponds to the number of hours spent on the topic of length for that particular grade.		

The activities in the Toolkit are to be mediated through lesson plans, with the number of lesson plans corresponding to the number of hours allocated for each grade.

However, it is our belief that teachers should have the responsibility and opportunity to design their own lesson plans, based on the kind of learners they have in their classes. For example, they have to cater for differentiations according to the needs of their learners. However, sample lesson plans are included in the Toolkit as guidance.

Other information from CAPS that features in the Toolkit relates to integration within mathematics, as well as across other subjects. For example, mental mathematics, which has to be incorporated into and done during each lesson when teaching length, belongs to the content area of Numbers, Operations and Relationships. CAPS lists the concepts and skills relating to mental mathematics for each sub-topic. Integration across other subjects is catered for in the Toolkit, with activities such as one on comparing and arranging lengths of different colour crayons. Suggestions are then given for further reinforcement when colours are taught during Life Skills lessons.

The *Guidelines for responding to learner diversity in the classroom through CAPS* document provides information to teachers on how to facilitate and support curriculum differentiation in the classroom. Analysis of this document reveals that teachers are expected to create and use flexibly their own innovative material, methods of presentation, learning activities, lesson organization and assessment (DBE, 2011b). The document outlines generic strategies from which teachers can base their own innovations. However, teachers lack mathematical content knowledge and skills needed to apply what they know in the classroom (Stols *et. al.*, 2015). Also, the training of PST's does not prepare them for rethinking their beliefs surrounding the teaching profession and adapting to new ways or strategies of mediating the content they teach (Lauwerier & Akkari, 2015).

The Toolkit on length provides PST's with specific strategies for responding to learner diversity in the context of each sub-topic presented. For example, learning material allows learners to engage with hands-on activities that provide for tactile and bodily-kinesthetic learning experiences (see Figure 2 below).

Introduction to sub-topic: Rounding off numbers in the context of length

Estimate the length of your hand span in centimetres. Remember that your hand span is the distance from the tip of your thumb across the palm, to the tip of your small finger, whey your hand is opened as wide as possible. Refer to the picture below:

Now use a ruler to measure your hand span in centimetres.

FIGURE 2: Activities involving tactile and bodily-kinesthetic learning experiences

Electronic material, in the form of Geogebra, is used in the Toolkit, to further respond to learner diversity, in addition to the use of geoboards, which is suggested in CAPS (DBE 2011a, p. 232).

The document on guidelines for the promotion of learner diversity stipulates that learning activities should include problem solving that allows learners to explore concepts rather than reproduction of prior knowledge (DBE, 2011b). The Toolkit on length provides specific activities for PST's to facilitate such problem solving activities with learners (see example in Figure 3 below).

Introduction to sub-topic: Measuring lengths practically and accurately, using a ruler

The class teacher gives new pencils to learners as soon as the length of the pencil becomes less than half of the original length of a new pencil. She does this so that learners do not struggle holding the pencil when using it. Let us look at Sino's and Angel's pencils. How can we find out if the teacher needs to replace the two learners' pencils or not?

This is a new pencil: 

This is Sino's pencil: 

This is Angel's pencil: 

FIGURE 3: An activity that promotes problem-solving

The purpose of the document on *Mathematics Teaching and Learning Framework for South Africa: Teaching mathematics for understanding* is "to attend to the challenges associated with the teaching and learning of mathematics, so that learner outcomes are improved" (DBE 2018, p. 3). Analysis of this document reveals that the framework is mainly embedded in the theory of mathematical proficiency, which proposes five strands for promoting mathematics understanding, namely, conceptual understanding, procedural fluency, strategic competence, adaptive reasoning and productive disposition (Kilpatrick *et. al.*, 2001). Only the first four of the strands (or dimensions) are applicable in the framework. Examples of how to apply the four dimensions are provided for each phase of schooling. Teachers are then left to design their own learning centred activities and methods. However, most teachers are not able to provide opportunities for learner-centred instruction (Lauwerier & Akkari, 2015), and they give little encouragement for learners to "predict, describe, justify, represent or construct mathematical knowledge" (Luneta 2015, p. 5).

In the Toolkit on length, suggestions are given for each activity on how to develop learners' mathematical understanding. The example below was designed to develop learners' conceptual understanding, procedural fluency and adaptive reasoning

involving the sub-topic: Converting between different units of length – mm, cm and m.

> During athletic sports day, learners participated in long jump. The lengths of their jumps were measured in metres, using a tape measure. The top (longest) two distances jumped belonged to Sethu and Ayanda, which were both recorded as 3 cm. The two athletes had something to say about this score:
>
> 1. Ayanda claimed that there is something wrong with this measurement. The athletics officials examined this claim and realized that they made an error. What error could this be?
> 2. After the error was corrected, Sethu argued that the scores could have been different if smaller units were used when taking the measurement. What units could those be and how could this have possibly made a difference in the recorded measurements of the distances?

Analysis of other Teaching and Learning Support Material (TLSM)

The South African DBE prescribes and recommends certain material to support the delivery of the curriculum. These are mainly the Workbooks for the different IP grades and some textbooks from various authors and publishers. An analysis of these books reveals that they do not fully support the development of learners' mathematical proficiency as discussed in the preceding section. For example, the instructions from the grade 4 Workbook, as seen in the examples given in an earlier section of this paper, and many other exercises in the Workbooks, do not consciously encourage learners to apply some strategic competence or adaptive reasoning. They mostly promote procedural fluency and some conceptual understanding.

Other textbooks, which are recommended by the DBE, do give considerable guidance for teaching concepts and skills on length in order to teach for mathematical understanding, often in the Teacher's Guide (Human *et. al.*, 2016). However, they do not provide much theoretical underpinning of such concepts and skills to empower PST's with deep mathematical knowledge (of the concept of length in this case). The Toolkit then closes this gap by including theoretical dimensions to the teaching and learning of length, such as the key concepts and big ideas that form the foundation for children's understanding of linear measurement (see Table 3 below). These key concepts are not sufficiently supported by traditional measurement instruction (Sarama & Clements, 2009).

TABLE 3: Key concepts in linear measurement (Sarama & Clements 2009, pp. 275 – 277)

Concept	Description
1. Understanding of the attribute	Lengths span fixed distances
2. Conservation	The understanding that as an object is moved, its length does not change.
3. Transitivity	The understanding that if the length of object X is equal to (or greater/less than) the length of object Y and object Y is the same length as (or greater/less than) object Z, then object X is the same length as (or greater/less than) object Z.
4. Equal partitioning	The mental activity of slicing up an object into same-sized length units.
5. Unit iteration	The length measure is constituted through iterating the same unit along the length of the object being measured, without gaps or overlaps.
6. Accumulation of distance and additivity	The understanding that as you iterate a unit along the length of an object and count the iteration, the number words signify the space covered by all units counted up to that point.
7. Origin	The notion that any point on a ratio scale can be used as the origin.
8. Relation between number and measurement	Understanding that the units of length measurement being 'counted' are continuous rather than discrete; the number is used to quantify this distance in real-world contexts; there is an inverse relationship between the size of the unit and the number of units in a given length measure.

Analysis of Mathematics Education curriculum for PST's

Some shortcomings of the pre-service mathematics teacher education in South Africa have been discussed in the section on the background to this study. In this section we highlight the inadequacy of the PST Mathematics Education curriculum to prepare PST's for the development of their own material. Hence the need to design Toolkits that will be used by the PST's and assist them in facilitating learners' mathematical understanding.

When they enter the teaching career, South African teachers do not have the necessary mathematics subject content knowledge (Van der Sandt, 2007). The poor mathematical background tends to make them rely on the textbooks and other materials, which they

have not developed or adapted themselves (Feza & Webb, 2005). However, textbooks tend to present or explain mathematical ideas and concepts in simplistic and routine-like ways without, for example, providing for multiple strategies to solutions (Luneta, 2015). The Toolkit on length, in contrast, has been consciously designed to provide opportunities for the development of strategic competence as one of the strands of mathematical proficiency.

CONCLUSION AND RECOMMENDATIONS

The South African CAPS for IP provides clear guidelines on the knowledge and skills that need to be developed in learners when teaching the concept of length. However, it lacks clarity on the big ideas and key concepts that teachers need to, firstly understand themselves, and then develop in the learners to which they teach measurement of length. The CAPS also does not adequately empower teachers with the kind of errors and misconceptions they can expect when teaching length to their learners. Other curriculum documents that are meant to support the CAPS also stipulate the outcomes which the teaching of mathematics, and length in the context of this paper, should achieve. These documents provide generic statements and expect teachers to use these to design their own learning experiences for developing learners' mathematical understanding. These expectations present a challenge, given that teachers and PST's are not adequately empowered with skills for designing their own material. It is also evident that available LTSM on teaching the topic of length to IP learners does not fully promote the development of mathematical proficiency in learners. Additional LTSM that is available to teachers and PST's also has some shortcomings in terms of providing them with deep mathematical knowledge that is required to teach the concept competently.

The development of the Toolkit on length is meant to bridge the gaps in the currently available material and provide adequate support for teaching the topic. The current Toolkit is a working document that still needs to be tested with B.Ed. IP PST's to determine the extent to which it develops their understanding of length and its theoretical underpinnings, as well as their pedagogical knowledge to allow them to mediate the content to learners. This paper has outlined the processes and steps involved in the development of the Toolkit on length. We recommend the steps, as summarized in Table 4 below, and hope that they would help PST's and other teachers to develop similar Toolkits.

TABLE 4: Steps involved in the development of a teaching and learning Toolkit

1. Analysis of curriculum documents	Identify the knowledge, concepts and skills that need to be developed regarding the topic of concern, from the main curriculum document. Determine and note any lack of clarity or omissions regarding the concepts and skills or how to facilitate learners' development of these. Check if there are other additional curriculum documents that support the main document; Identify their purpose, focus, expected outcomes, their relevance to the topic concerned and any shortcomings.
2. Analysis of other available LTSM	Explore several LTSM and determine the extent to which they facilitate the development of the concepts and skills stipulated in the curriculum documents; note any shortcomings.
3. Analysis of Mathematics Education curriculum for PST's.	Find literature that documents Mathematics Education curriculum for the PST's to determine if it is adequate in preparing them for mediating the teaching and learning of the concepts and skills applicable to the topic, given the available resources. Include theoretical aspects of the topic.
4. Development of the Toolkit	Using all of the above as guidance, develop the Toolkit, and address any shortcomings identified; this becomes the working document that will need to be evaluated, modified and strengthened as the case might be.

The above steps are not cast in stone; they provide for flexibility in application and are meant to provide a possible pathway to follow when developing similar Toolkits.

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REFERENCES

Ball, D., Hill, H. & Bass, H. (2005). Knowing mathematics for teaching: Who knows mathematics well enough to teach third grade, and how can we decide. *American Educator, 29*(1), 14-17, 20-22, 43-46.

Ball, D. L., Thames, M. H., & Phelps, G. (2008). Content knowledge for teaching: what makes it special? *Journal of Teacher Education, 59*(5), 389-407.

Bernstein, A. (Ed.). (2013). *Mathematics outcomes in South African schools. What are the facts? What should be done?* Report published by The Centre for Development and Enterprise.

Charalambous, Y. (2010). Mathematical knowledge for teaching and task unfolding: An Exploratory Study, *The Elementary School Journal, 110*(3), 247-278.

Clements, D. H. (1999). Teaching length measurement: Research challenges. *School Science and Mathematics, 99*(1), 4–11.

Department of Basic Education [DBE] (2011a). *Curriculum Assessment Policy Statement, Grades 4 -6. Mathematics*. Pretoria: Department of Basic Education.

Department of Basic Education [DBE] (2011b). *Guidelines for responding to learner diversity in the classroom through Curriculum and Assessment Policy Statements*. Pretoria: Department of Basic Education.

Department of Basic Education [DBE] (2018). *Mathematics Teaching and Learning Framework for South Africa: Teaching mathematics for understanding.* Pretoria: Department of Basic Education.

Department of Basic Education [DBE] (2019). *Mathematics in English Grade 4, Book 1, Terms 1 & 2 (9th ed.)*. Pretoria: Department of Basic Education.

Feza, N., & Webb, P. (2005). Assessment standards, van Hiele levels, and grade seven learners' understandings of geometry. *Pythagoras*, (62), 36-47.

Fonseca, K., Maseko, J. & Roberts, N. (2018). Students' mathematical knowledge in a Bachelor of Education (Foundation or Intermediate Phase) programme. In Govender, R. & Junqueira, K., *Proceedings of the 24th Annuals National Congress of the Association for Mathematics Education of South Africa (AMESA)* ISBN: 978-0-6399514-0-9, AMESA 2018 Long Paper, 25–29 June 2018. University of the Free State, Bloemfontein Campus, Free State, pp. 124–139.

Graven, M., & Vent, H. (2017). *Improving primary mathematics teaching and learning: Research for development in resource-constrained contexts*. London: McMillan Publishers.

Graven, M., Hexane, D., & Stott, D. (2013). The evolution of an instrument for researching young mathematical dispositions. African Journal of Research in Mathematics, Science and Technology Education, 17, 26–37.

Hansen, A. (2017). *Children's errors in mathematics (4th Ed.)*. London: SAGE Publications.

Hill, H. C., Blank, M. L., Charalambous, C. Y., Lewis, J. M., Phelps, G. C., Sleep, L. & Ball, D. L. (2008). Mathematical knowledge for teaching and the mathematical quality of instruction: An exploratory study, *Cognition and Instruction, 26*(4), 430-511.

Human, P., Olivier, A., Lampen, E., Le Roux, A., Long, C., & Human, C. (2016). Mathematics Grade 4: Teacher Guide. Rietondale: The Ukuqonda Institute.

Kilpatrick, J., Swafford, J. & Findell, B. (Eds.). (2001). *Adding it up: Helping children learn mathematics*. Washington, DC: National Academy Press.

Lauwerier, T. & Akkari, A. (2015). Teachers and the quality of basic education in Sub-Saharan Africa. *Education Research and Foresight: Working Papers Series, 2015, no. 1.*

Lehrer, R. (2003). Developing understanding of measurement. In J. Kilpatrick, W. Martin & D. Schifter (Eds.), *A research companion to principles and standards for school mathematics* (pp. 179–192). Reston, VA: NCTM.

Luneta, K. (2014). Foundation Phase student teachers' knowledge of Space and Shape. *South African Journal of Childhood Education, 4*(3), 71-86.

Luneta, K. (2015). Issues in communicating mathematically in rural classrooms in South Africa. *Journal of Communication, 6*(1), 1-9.

Modisaotsile, B. M. (2012). The failing standard of basic education in South Africa. *African Institute of South Africa, 72*, 1 – 8.

PrimTEd Newsletter. (2016, October). Newsletter 1. Retrieved from
https://www.jet.org.za/clearinghouse/primted/newsletter/primted-newsletter-1-october-2016.pdf

Rowland, T. & Ruthven, K. (Eds.) 2011. *Mathematical knowledge in teaching*. New York: Springer.

Ryan, J. & Williams, J. (2007). *Children's mathematics 4–15: Learning from errors and misconceptions*. Maidenhead, England: Open University Press.

Sarama, J. & Clements, D. H. (2009). *Early childhood mathematics education research: Learning trajectories for young children*. England: Taylor & Francis.

Solomon, T. L. & Vasilyeva, M. (2015). Minding the gap: Children's difficulty conceptualising spatial linear measurement units. *Developmental Psychology, 51*(11), 1564–1573.

Spaull, N. (2013). South Africa's education crisis: The quality of education in South Africa 1994-2011. Report commissioned by Centre for Development and Enterprise.

Stols, G., Ferreira, R., Pelser, A., Olivier, W. A., Van der Merwe, A., De Villiers, C. & Venter, S. (2015). Perceptions and needs of South African Mathematics teachers concerning their use of technology for instruction. *South African Journal of Education,35*(4), 1209 – 1221.

Suggate, J., Davis, A., & Goulding, M. (2010). *Mathematical knowledge for primary teachers (4th ed.)*. London: Routledge.

Tan-Sisman, G. & Aksu, M. (2016). A study on sixth grade students' misconceptions and errors in spatial measurement: Length, area and volume. International Journal of Science and Mathematics Education, 14(7), 1293–1319.

Van der Sandt, S. (2007). Pre-service geometry education in South Africa: a typical case? *IUMPST: The Journal, 1*, 1-9.

Van de Walle, J. A., Karp, K. S., & Bay-Williams, J.M. (2015). *Elementary and middle school mathematics: Teaching developmentally*. Pearson Education.